Chapter 6 Review Solutions

Geometry
Chapter 6 - Chapter Review Solutions

1. A parallelogram with four congruent sides is a *rhombus*.

2. A polygon with all angles congruent is an *equiangular polygon*.

3. Angles of a polygon that share a side are *consecutive*.

4. A *trapezoid* is a quadrilateral with exactly one pair of parallel sides.

5. By the Corollary to the Polygon Angle-Sum Theorem,

$$\begin{aligned}\text{Measure} &= \frac{(n-2)180}{n}\\ &= \frac{(6-2)180}{6}\\ &= \frac{4\cdot 180}{6}\\ &= 120\end{aligned}$$

Use the Polygon Exterior Angle-Sum Theorem to find the measure of an exterior angle.

$$\begin{aligned}\text{Measure} &= \frac{360}{n}\\ &= \frac{360}{6}\\ &= 60\end{aligned}$$

6. By the Corollary to the Polygon Angle-Sum Theorem,

$$\begin{aligned}\text{Measure} &= \frac{(n-2)180}{n}\\ &= \frac{(16-2)180}{6}\\ &= \frac{14\cdot 180}{16}\\ &= 157.5\end{aligned}$$

Use the Polygon Exterior Angle-Sum Theorem to find the measure of an exterior angle.

$$\begin{aligned}\text{Measure} &= \frac{360}{n}\\ &= \frac{360}{16}\\ &= 22.5\end{aligned}$$

7. By the Corollary to the Polygon Angle-Sum Theorem,

$$\begin{aligned}\text{Measure} &= \frac{(n-2)180}{n}\\ &= \frac{(5-2)180}{5}\\ &= \frac{3\cdot 180}{5}\\ &= 108\end{aligned}$$

Use the Polygon Exterior Angle-Sum Theorem to find the measure of an exterior angle.

$$\begin{aligned}\text{Measure} &= \frac{360}{n}\\ &= \frac{360}{5}\\ &= 72\end{aligned}$$

8. The sum of the measures of the exterior angles is 360, in exercises 6-8, as stated by the Polygon Exterior Angle-Sum Theorem.

9. By the Polygon Angle-Sum Theorem, the sum of the measures of the interior angles of a pentagon is $(5-2)180 = 3\cdot 180 = 540$. So,

$$\begin{aligned}83 + 90 + 119 + 89 + x &= 540\\ x &= 540 - 381\\ x &= 159\end{aligned}$$

10. By the Polygon Angle-Sum Theorem, the sum of the measures of the interior angles of a quadrilateral is $(4-2)180 = 2\cdot 180 = 360$. So,

$$\begin{aligned}122 + 90 + 79 + z &= 360\\ z &= 360 - 291\\ z &= 69\end{aligned}$$

11. Since alternate interior angles of $\parallel$ line are $\cong$, $m\angle 1 = 38$. By Theorem 6-5, $m\angle 3 = 99$. Using the Triangle Angle-Sum Theorem, $m\angle 2 + 38 + 99 = 180$; $m\angle 2 = 180 - 137 = 43$.

12. By Theorem 6-5, opposite angles are $\cong$ so $m\angle 2 = 79$. Consecutive angles are $\cong$ by Theorem 6-4. Thus $m\angle 1 = 180 - 79 = 101$, and $m\angle 3 = m\angle 1 = 101$ by Theorem 6-5 again.

13. Since alternate interior angles of $\parallel$ lines are $\cong$, $m\angle 1 = 37$. By the Triangle Angle-Sum Theorem,

$$\begin{aligned}m\angle 3 + 37 + (180 - 63) &= 180\\ m\angle 3 &= 63 - 37\\ m\angle 3 &= 26\end{aligned}$$

Since alternate interior angles of $\parallel$ are $\cong$, $m\angle 2 = m\angle 3 = 26$.

14. Since the 4 triangles are right isosceles triangles, the acute base angels all have the measure $\frac{180-90}{2}=45$ by the Triangle Angle-Sum Theorem. Thus, $m\angle 1 = m\angle 2 = m\angle 3 = 45$.

15. By Theorem 6-3, opposite sides are $\cong$. Write the system of equations:

$$\begin{array}{lcl} AB = CD & & BC = DA \\ 2y = 5x - 1 & \text{and} & y + 3 = 2x + 4 \\ y = \frac{1}{2}(5x-1) & & y = 2x + 1 \end{array}$$

Substitute $y = 2x + 1$ for y in the first equations and solve for x.

$$2x + 1 = \frac{1}{2}(5x-1)$$
$$4x + 2 = 5x - 1$$
$$2 + 1 = 5x - 4x$$
$$x = 3$$

So, $y = 2(3) + 1 = 6 + 1 = 7$.

16. By Theorem 6-3, opposite sides are $\cong$. Write the system of equations:

$$\begin{array}{lcl} AB = CD & & BC = DA \\ 2y + 1 = 7x - 3 & \text{and} & y + 1 = 3x \\ y = \frac{1}{2}(7x-4) & & y = 3x - 1 \end{array}$$

Substitute $y = 3x - 1$ for y in the first equations and solve for x.

$$3x - 1 = \frac{1}{2}(7x-4)$$
$$6x - 2 = 7x - 4$$
$$-2 + 4 = 7x - 6x$$
$$x = 2$$

So, $y = 3(2) - 1 = 6 - 1 = 5$.

17. No; there is no information about the diagonals, angles, or sides. Vertical angles are $\cong$ for all lines.

18. Yes; by Theorem 6-11.

19. By Theorem 6-9, $ABCD$ is a $\square$ if consecutive angles are supplementary. So,

$$\begin{aligned} m\angle B + m\angle C &= 180 \\ (3y-20)+(4y+4) &= 180 \\ 7y-16 &= 180 \\ y &= \frac{196}{7} \\ y &= 28 \end{aligned}$$

and

$$\begin{aligned} m\angle A + m\angle D &= 180 \\ 4x+(2x+6) &= 180 \\ 6x+6 &= 180 \\ x &= \frac{174}{6} \\ x &= 29 \end{aligned}$$

20. By Theorem 6-11, $ABCD$ is a $\square$ if diagonals bisect each other. Write the system of equations

$$\begin{aligned} 4x-2 &= 3y-1 \\ 4x &= 3y-1+2 \\ x &= \frac{1}{4}(3y+1) \end{aligned} \quad \text{and} \quad \begin{aligned} 3y-3 &= 3x \\ \frac{3y-3}{3} &= x \\ y-1 &= x \end{aligned}$$

Substitute $x = y-1$ in the first equation and solve for y.

$$\begin{aligned} y-1 &= \frac{1}{4}(3y+1) \\ 4y-4 &= 3y+1 \\ 4y-3y &= 1+4 \\ y &= 5 \end{aligned}$$

Thus, $x = 5-1 = 4$.

21. Since alternate interior angles of $\parallel$ lines are $\cong$, $m\angle 2 = 32$. By Theorem 6-13, diagonals of a rhombus are $\perp$. So, $m\angle 3 = 90$. By the Triangle Angle-Sum Theorem,

$$\begin{aligned} m\angle 1 + 32 + 90 &= 180 \\ m\angle 1 &= 180-122 \\ m\angle 1 &= 58 \end{aligned}$$

22. $m\angle 1 = 180-56 = 124$. Since diagonals of a $\square$ bisect each other by Theorem 6-6, and the diagonals of a rectangle are $\cong$ by Theorem 6-15, the triangles are isosceles triangles. Thus, by the Triangle Angle-Sum Theorem, $m\angle 2 = \frac{180-124}{2} = \frac{56}{2} = 28$ and $m\angle 3 = \frac{180-56}{2} = \frac{124}{2} = 62$.

23. Sometimes; a rhombus is a square when the angles are right angles.

24. Always; a square is always a rectangle since it has right angles.

25. Sometimes; a rhombus is a rectangle when it has right angles, namely when it is a square.

26. Sometimes; the diagonals of a ▱ are $\perp$ when it is a rhombus (Theorem 6-13).

27. Sometimes; the diagonals of a ▱ are $\cong$ when it is a rectangle (Theorem 6-15).

28. Always; opposite angles of a ▱ are always $\cong$ (Theorem 6-5).

29. No; the $\cong$ alternate interior angles tell you that the sides are $\parallel$, but two sides are $\parallel$ for all parallelograms.

30. Yes; the ▱ is a rhombus because opposite sides of a ▱ are $\cong$ by Theorem 6-3. Since ▱ is also a rectangle, it must be a square.

31. The diagonals of a rhombus bisect a pair of opposite angles by Theorem 6-14. So,

$$5x - 30 = 3x + 6$$
$$5x - 3x = 6 + 30$$
$$x = \frac{36}{2}$$
$$x = 18$$

32. Since diagonals of a ▱ bisect each other by Theorem 6-6, and the diagonals of a rectangle are $\cong$ by Theorem 6-15,

$$2x - 1 = x + 3$$
$$2x - x = 3 + 1$$
$$x = 4$$

33. Since same-side interior angles of $\parallel$ lines are supplementary, $m\angle 1 = 180 - 45 = 135$. By Theorem 6-19, base angles of an isosceles trapezoid are $\cong$. So, $m\angle 3 = 45$ and $m\angle 2 = m\angle 1 = 135$.

34. Since same-side interior angles of $\parallel$ lines are supplementary, $m\angle 3 = 180 - 80 = 100$. By Theorem 6-19, base angles of an isosceles trapezoid are $\cong$. So, $m\angle 1 = 80$ and $m\angle 2 = m\angle 3 = 100$.

35. The diagonals of a kite are $\perp$ by Theorem 6-22. So, $m\angle 1 = 90$. By the Triangle Angle-Sum Theorem, $m\angle 2 = 180 - (90 + 65) = 180 - 155 = 25$.

36. The diagonals of a kite are $\perp$ by Theorem 6-22. By the Triangle Angle-Sum Theorem, $m\angle 1 = 180 - (90 + 34) = 180 - 124 = 56$ and $m\angle 2 = 180 - 34 - 56 - 38 = 52$.

37. Use the Trapezoid Midsegment Theorem,

$$\begin{aligned} 5x - 3 &= \frac{1}{2}[(6x - 1) + 3] \\ 10x - 6 &= 6x + 2 \\ 10x - 6x &= 2 + 6 \\ 4x &= 8 \\ x &= 2 \end{aligned}$$

38. Scalene; by counting, the lengths of two sides are 3 units and 4 units. The third side has to be longer, so $\triangle ABC$ is scalene.

39. Isosceles; find the lengths of the sides using the Distance Formula.

$$\begin{aligned} \text{side } 1 &= \sqrt{(1-0)^2 + (-1-2)^2} \\ &= \sqrt{1+9} \\ &= \sqrt{10} \\ \text{side } 2 &= \sqrt{(3-0)^2 + (3-2)^2} \\ &= \sqrt{9+1} \\ &= \sqrt{10} \\ \text{side } 3 &= \sqrt{(3-1)^2 + (3-(-1))^2} \\ &= \sqrt{4+16} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \end{aligned}$$

Since the 2 lengths are the same, the triangle is isosceles.

40. $\square$; find the lengths of the sides using the Distance Formula.

$$\begin{aligned}GR &= \sqrt{(5-2)^2+(8-5)^2}\\ &= \sqrt{9+9}\\ &= \sqrt{18}\\ &= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}RA &= \sqrt{(-2-5)^2+(12-8)^2}\\ &= \sqrt{49+16}\\ &= \sqrt{65}\end{aligned}$$

$$\begin{aligned}AD &= \sqrt{(-5-(-2))^2+(9-12)^2}\\ &= \sqrt{9+9}\\ &= \sqrt{18}\\ &= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}DG &= \sqrt{(-5-2)^2+(9-5)^2}\\ &= \sqrt{49+16}\\ &= \sqrt{65}\end{aligned}$$

Since $\overline{GR} \cong \overline{AD}$ and $\overline{RA} \cong \overline{DG}$, $GRAD$ is a $\square$ by Theorem 6-8. Use the Slope Formula to find the slopes of the consecutive sides: slope of $\overline{GR} = \frac{8-5}{5-2} = \frac{3}{3} = 1$; slope of $\overline{RA} = \frac{12-8}{-2-5} = -\frac{4}{7}$. Since the product of the slopes is not -1, there are no right angles. Thus, $GRAD$ is a only a $\square$.

41. Kite; find the lengths of the sides using the Distance Formula.

$$\begin{aligned}FI &= \sqrt{(1-(-13))^2+(12-7)^2}\\ &= \sqrt{196+25}\\ &= \sqrt{221}\end{aligned}$$

$$\begin{aligned}IN &= \sqrt{(15-1)^2+(7-12)^2}\\ &= \sqrt{196+25}\\ &= \sqrt{221}\end{aligned}$$

$$\begin{aligned}NE &= \sqrt{(1-15)^2+(-5-7)^2}\\ &= \sqrt{196+144}\\ &= \sqrt{344}\end{aligned}$$

$$\begin{aligned}EF &= \sqrt{(7-(-5))^2+(-13-1)^2}\\ &= \sqrt{144+196}\\ &= \sqrt{344}\end{aligned}$$

Since two pairs of consecutive sides are $\cong$ and no opposite sides are $\cong$, $FINE$ is a kite.

42. Rhombus; find the lengths of the sides using the Distance Formula.

$$QU = \sqrt{(12-4)^2 + (14-5)^2} = \sqrt{64+81} = \sqrt{145}$$

$$UA = \sqrt{(20-12)^2 + (5-14)^2} = \sqrt{64+81} = \sqrt{145}$$

$$AD = \sqrt{(12-20)^2 + (-4-5)^2} = \sqrt{64+81} = \sqrt{145}$$

$$DQ = \sqrt{(4-12)^2 + (5-(-4))^2} = \sqrt{64+81} = \sqrt{145}$$

Since $\overline{QU} \cong \overline{AD}$ and $\overline{UA} \cong \overline{DQ}$, $QUAD$ is a ▱ by Theorem 6-8. Use the Slope Formula to find the slopes of the consecutive sides: slope of $\overline{QU} = \frac{14-5}{12-4} = \frac{9}{8}$; slope of $\overline{UA} = \frac{5-14}{20-12} = -\frac{9}{8}$. Since the product of the slopes is not -1, there are no right angles. So, $QUAD$ is a rhombus since $\overline{QU} \cong \overline{UA} \cong \overline{AD} \cong \overline{DQ}$.

43. Isosceles trapezoid; $\overline{HA} \parallel \overline{TW}$ since both segments are horizontal. Find the lengths of the legs using the Distance Formula.

$$WH = \sqrt{(-9-(-11))^2 + (10-4)^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$$

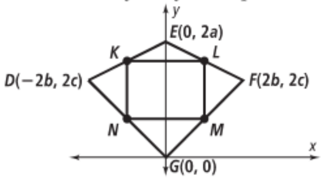
$$AT = \sqrt{(4-2)^2 + (4-10)^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$$

Since $\overline{WH} \cong \overline{AT}$, $WHAT$ is an isosceles trapezoid.

44. Since $SL = 2a$ and the rhombus $FLPS$ is center at the origin, points S and L are a units away from the origin on the x-axis. Since $FP = 4b$, F and P are $2b$ units away from the origin on the y-axis. Thus, the coordinates of the vertices are $F(0,2b)$, $L(a,0)$, $P(0,-2b)$, and $S(-a,0)$.

45. Since opposite sides of a ▱ are $\cong$ and $\parallel$, the y-coordinate of P is c and the x-coordinate is $a-b$. So, the coordinates of P are $P(a-b,c)$

46. Answers may vary. Sample:



Given: Kite $DEFG$, K, L, M, N are midpoints of sides

Prove: $KLMN$ is a rectangle.

Use the Midpoint Formula to find the coordinates of the midpoints:

$$K = \left(\frac{0+(-2b)}{2}, \frac{2a+2c}{2}\right) = (-b, a+c)$$

$$L = \left(\frac{0+2b}{2}, \frac{2a+2c}{2}\right) = (b, a+c)$$

$$M = \left(\frac{2b+0}{2}, \frac{2c+0}{2}\right) = (b, c)$$

$$N = \left(\frac{-2b+0}{2}, \frac{2c+0}{2}\right) = (-b, c)$$

By the Slope Formula,

slope of $\overline{KL} = \dfrac{(a+c)-(a+c)}{b-(-b)} = 0$

slope of $\overline{LM} = \dfrac{c-(a+c)}{b-b} =$ undefined

slope of $\overline{NM} = \dfrac{c-c}{-b-b} = 0$

slope of $\overline{KN} = \dfrac{(a+c)-c}{-b-(-b)} =$ undefined

So, $\overline{KL} \parallel \overline{NM}$ and $\overline{KN} \parallel \overline{LM}$, and thus $KLMN$ is a ▱ by definition of ▱. Since horizontal and vertical lines are $\perp$, $\overline{KL} \perp \overline{LM}$, and $KLMN$ is a rectangle.