Geometry Section 8-6 Solutions

Geometry
Lesson 8-6 - Practice and Problem-Solving Exercises Solutions

15. Let p be the measure of the angle with its vertex at the pitcher.
Use the Law of Cosines.
$$90^2 = 57^2 + 110^2 - 2(57)(110)\cos p$$
Solve for p.
$$8100 = 3249 + 12{,}100 - 12{,}540\cos p$$
$$\frac{-7249}{-12{,}540} = \cos p$$
$$p = \cos^{-1}\left(\frac{-7249}{-12{,}540}\right)$$
$$p \approx 54.68507678$$
The angle measure is about 54.7°.

16. Let r be the measure of the angle where the sides of the ravine meet.
Use the Law of Cosines.
$$20^2 = 12^2 + 14^2 - 2(12)(14)\cos r$$
Solve for r.
$$400 = 144 + 196 - 336\cos r$$
$$\frac{60}{-336} = \cos r$$
$$r = \cos^{-1}\left(\frac{60}{-336}\right)$$
$$r \approx 100.2865606$$
The angle measure is about 100.3°.

17. Before you can solve this problem, you need to find the length of the third side. Use the Law of Cosines. Drawing a diagram can help so that you can see the correct correspondences between the angles and sides.
Let x be the length of the third side.
$$x^2 = 32^2 + 39^2 - 2(32)(39)\cos 76°$$
$$x \approx 44.05863081$$
The third side is about 44 feet.
Add the three sides to get 115 ft.

18. Let d be the distance for the last leg of the trip.
$$d^2 = 57^2 + 90^2 - 2(57)(90)\cos 60°$$
$$d \approx 78.86063657$$
The distance is about 78.9 mi.

19. Law of Sines;

$$\frac{\sin 40°}{6} = \frac{\sin x}{8}$$

$$\sin x = \frac{8\ \sin 40°}{6}$$

$$x = \sin^{-1}\left(\frac{8\ \sin 40°}{6}\right) \approx 58.98696953$$

x is about 59.0.

20. Law of Cosines;

$$x^2 = 7^2 + 10^2 - 2(7)(10)\cos\ 48°$$

$$x \approx 7.43785689$$

x is about 7.4.

21. Law of Sines;

$$\frac{\sin 110°}{x} = \frac{\sin 40}{12}$$

$$x = \frac{12\ \sin 110°}{\sin 40}$$

$$x \approx 17.5428264$$

x is about 17.5.

22. Law of Cosines;

$$x^2 = 23^2 + 27^2 - 2(23)(27)\cos\ 36°$$

$$x \approx 15.91228748$$

x is about 15.9.

23. Let x be the distance.
Use the Law of Cosines.

$$x^2 = 9.5^2 + 15^2 - 2(9.5)(15)\cos\ 95°$$

$$x \approx 18.44151259$$

x is about 18.4 ft.

24. Let f be the measure of the angle where the shorter sides meet.
Use the Law of Cosines.

$$27^2 = 18^2 + 25^2 - 2(18)(25)\cos f$$

Solve for f.

$$729 = 324 + 625 - 900\cos f$$

$$\frac{-220}{-900} = \cos f$$

$$f = \cos^{-1}\left(\frac{-220}{-900}\right)$$

$$f \approx 75.85099549$$

The angle is about 75.9.

25. In a parallelogram, the opposite sides are equal. Solve for x.

$$2(2x + 3x - 3) = 62$$
$$10x - 6 = 62$$
$$10x = 68$$
$$x = 6.8$$

Substitute 6.8 for x to get the side lengths of the parallelogram.
2(6.8) = 13.6 and 3(6.8) – 3 = 17.4.
The sides of the parallelogram are 13.6 and 17.4.
Use the Law of Cosines.

$$TR^2 = 13.6^2 + 17.4^2 - 2(13.6)(17.4)\cos\ 120°$$
$$TR \approx 26.91393691$$

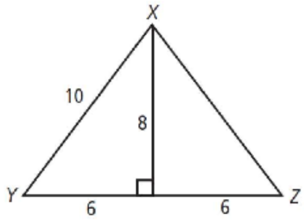
TR is about 26.9 mm.

26. If the angle of elevation to the base of the monument is 11° and the angle of elevation to the top of the monument is 26°, subtract to find the angle of the triangle including the height of the monument, or 15°. Let x be the height of the monument.
Use the Law of Cosines.

$$x^2 = 12.4^2 + 13.3^2 - 2(12.4)(13.3)\cos\ 15°$$
$$x \approx 3.471170618$$

The height is about 3.5 m.

27. In an isosceles triangle, the height from the vertex to the base bisects the base and is perpendicular to it. So, the height divides the triangle into two congruent right triangles with height 8 and base $= \frac{1}{2}(12) = 6$.



Use the Pythagorean Theorem to find the length of the leg of the isosceles triangle, the hypotenuse of one of the triangles.
Let h = length of hypotenuse.

$$\begin{aligned} 6^2 + 8^2 &= h^2 \\ 36 + 64 &= h^2 \\ 100 &= h^2 \\ \sqrt{100} &= h \\ 10 &= h \end{aligned}$$

The length of a leg of the isosceles triangle, the hypotenuse of one of the triangles, is 10.

Use the Law of Sines. Let X be the vertex angle and Y be one base angle.

$$\frac{\sin Y}{y} = \frac{\sin 90^\circ}{h}$$

$$\frac{\sin Y}{8} = \frac{\sin 90^\circ}{10}$$

$$\sin Y = \frac{8 \sin 90^\circ}{10}$$

$$Y = \sin^{-1}\left(\frac{8 \sin 90^\circ}{10}\right)$$

$m\angle Y \approx 53.13010235$
$m\angle Y \approx 53.1^\circ$

So, because the base angles of an isosceles triangle are congruent, each base angle is about 53.1°. Use the Triangle Sum Theorem to find $m\angle X$. Do not round $m\angle Y$ to find $m\angle X$.

$m\angle X = 180 - 53.13010235 - 53.13010235 = 73.7397953$
$m\angle X \approx 73.7^\circ$

So $m\angle X \approx 73.7^\circ$, $m\angle Y \approx 53.1^\circ$, $m\angle Z \approx 53.1^\circ$.